alism appears to be quite sensitive to assumptions made for the final gamma decay, in which spin states decay to the isomeric state for which the change in angular momentum is a minimum. As a consequence, the value for $\sigma$ would depend strongly on the spin values of the final states. For example, if the spins of the $Sc^{44}$ isomers had been established as 7 and 3 (instead of the now recognized values of 6 and 2), the $\sigma$ value would have been closer to 3.

To summarize, we have seen that the Huizenga and Vandenbosch formalism can adequately describe the isomer ratios for a compound-nucleus $(\alpha,n)$ reaction when interference from the $(\alpha,2n)$ is not possible. When multiple-particle emission becomes possible, however, the cross sections are strongly governed by the channel fraction parameters. The indiscriminate application of the Huizenga and Vandenbosch theory to even the most simple compound-nucleus reactions can be erroneous and yield parameters which are not meaningful once multiple-particle emission becomes possible.

### ACKNOWLEDGMENTS

We are grateful to the U. S. Atomic Energy Commission for providing financial support for this research and to the U. S. Air Force for support of the FSU Tandem van de Graaff Program. We also wish to thank Peter Stoycheff for helping with the operation of the accelerator.

PHYSICAL REVIEW VOLUME 135, NUMBER 6B 21 SEPTEMBER 1964

# Effect of the Harwell $A(\theta)$ Data on the 50-MeV Proton-Proton Phase Shifts*

PETER SIGNELL
*Physics Department, The Pennsylvania State University, University Park, Pennsylvania*
(Received 7 May 1964)

The new $A(\theta)$ and $\sigma(90°)$ data at 50 MeV are found to decrease the probable range of the phase shifts. The $^3F_3$ phase, however, is strongly predicted to be far from the value expected on the basis of models; this reflects on the consistency of the data. The most significant result is the rejection of solution 2. Comparison is made to the results of Batty and Perring.

## I. INTRODUCTION

In a previous communication[1] energy-independent modified phase-shift analyses were made of 25 proton-proton scattering data measured at energies near 50 MeV; these included cross-section, polarization, depolarization, and correlation measurements. The analysis energy was chosen to be 51.8 MeV, the energy of the data subgroup with the smallest quoted errors. An interesting result was that the $^3P_0$ phase shift was found to probably lie between 13 and 19°, somewhat higher than the 10.7–12.0° given by current potential models.

There has recently become available from Ashmore *et al.*[2] a set of quite good $A(\theta)$ measurements at 47.5 MeV, and from Batty *et al.*[3] a much improved absolute cross-section measurement at 50 MeV. The principal effects of these measurements are (1) that the probable $^3P_0$ phase-shift range is halved and lowered, and (2) that solution 2 is eliminated, for all practical purposes, at this energy.

## II. DATA SELECTION AND TREATMENT

Including the 25 data previously considered, 5 new $A(\theta)$ data, and a new absolute cross-section measurement at 90°, there are available a total of 31 data in the energy range 47.5–52.0 MeV. One has then to decide upon the energy at which to make the analysis.

The previous analysis[1] was performed at 51.8 MeV, the energy of the (then) most precise data. The new $A$ measurements, however, are at 47.5 MeV, and there are no $A$ data at nearby energies for use in interpolation. If the interpolation is not too large, one might consider using the results of an energy-dependent phase-shift analysis. The published phase-shift representation which appears to give the best fit to the moderate energy proton-proton data would seem to be that labeled "CR21" in a previous communication.[4] Using the CR21-predicted $A(\theta)$ at 47.5, 50, and 51.8 MeV, each experimental $A$ datum was shifted by the difference of the predictions at the datum angle. For example, CR21 predicted $A(39°)=-0.051$ at 47.5 MeV and

TABLE I. Interpolated $A(\theta)$ minus experimental $A(\theta)$ values, as fractions of the experimental errors. The 50-MeV numbers can be constructed from Table II.

| Energy (MeV) | c.m. angles (deg) 23.5 | 39.0 | 54.6 | 71.1 | 87.1 |
|---|---|---|---|---|---|
| 50 | −0.37 | −0.47 | −0.68 | −0.58 | −0.38 |
| 51.8 | −0.64 | −0.82 | −1.17 | −1.00 | −0.66 |

* Supported in part by the U. S. Atomic Energy Commission.
[1] P. Signell, N. R. Yoder, and N. M. Miskovsky, Phys. Rev. **133**, B149 (1964).
[2] Table II, Ref. a.
[3] Table II, Ref. c.
[4] P. Signell and N. R. Yoder, Phys. Rev. **134**, B100 (1964).

$-0.067$ at 50 MeV. Then the experimental $A(39°) = -0.020\pm0.034$ was shifted to $-0.036\pm0.034$. The shift in each datum, relative to its experimental standard deviation, is displayed in Table I. Although current potential models and CR21 yield a substantial spread in predicted $A(\theta)$ curves at one energy, the differences among the models were found to be insignificant for the shifts from 47.5 to 50 MeV. However, some model dependence developed by 51.8 MeV. In addition, note in Table I the sizable shifts to 51.8 MeV for several of the data. With the above in mind, together with the fact that the experimental energy of the very accurate $\sigma(90°)$ datum and of several other data was 50 MeV, the latter was chosen as the analysis energy.

The cross section angular distributions[1] were measured at 51.5 and 51.8 MeV, so their CR21 predictions at those energies and 50 MeV were examined. The *shape* changed by less than 0.2% for the large angle data, by less than 0.3% for the small angle data. Since the experimental errors are an order-of-magnitude larger than that, interpolation of the shapes was not necessary. The two normalizations were treated as completely unknown: They could have been interpolated, but their accuracy was considerably less than that of the new $\sigma(90°)$ with which they would compete. Those of the remaining 29 data which have been interpolated, or added, or changed since the previous analysis, are shown in Table II.

## III. ANALYSIS RESULTS

Phase-shift analyses were made of the 29 data at 50 MeV, with the higher angular momentum phases fixed at the CR21 50-MeV values, and the lower angular momentum phases adjusted so as to obtain a least-squares fit to the data. The order of release of the phases from their CR21 values was on the basis of a previously described goodness-of-fit criterion.[5] The results, shown in Table III, were unexpected in that the sixth released phase was strongly chosen to be $^3F_3$, and that the latter *changed sign* (Table IV). At the same time, the contribution to $\chi^2$ of $A(23.5°)$ dropped from 8.6 to 3.7. However, removal of this datum did not significantly alter the strong selection of $^3F_3$ as the sixth released phase, and $\chi^2$ now dropped from 26.5 to 18.1 when $^3F_3$ was released. This decrease was distributed among most of the data. We note that on the basis of Chauvenet's criterion, one would not expect to find a datum with a $\chi^2$ contribution of 7 or larger in a 29-piece data set.[5] No datum came close to Chauvenet's limit, except $A(23.5°)$ which slightly exceeded it for $N=5$.

TABLE II. Interpolated, revised, and added data (see text). The cross section is in mb/sr.

| Experimental energy (MeV) | c.m. angle (deg) | Type | Value at exptl. energy | Interpolated value (50 MeV) | Error | Reference |
|---|---|---|---|---|---|---|
| 47.5 | 23.5 | $A$ | −0.070 | −0.081 | 0.030 | a |
| | 39.0 | | −0.020 | −0.036 | 0.034 | |
| | 54.6 | | −0.009 | −0.026 | 0.025 | |
| | 71.7 | | 0.087 | 0.073 | 0.024 | |
| | 87.1 | | 0.168 | 0.160 | 0.021 | |
| 50.0 | 70.0 | $D$ | −0.241 | | 0.075 | b |
| 50.0 | 90.0 | $\sigma$ | 8.34 | | 0.05 | c |
| 50.0 | 45.0 | $P$ | 0.0316 | | 0.0017 | d |

[a] A. Ashmore, M. Devine, B. Hird, J. Litt, W. H. Range, M. E. Shepherd, and R. L. Clarke (private communication); submitted to Paris Conference, July 1964 (unpublished).
[b] T. C. Griffith, D. C. Imrie, G. J. Lush, and A. J. Metheringham, Phys. Rev. Letters **19**, 444 (1963), and T. C. Griffith *et al.* and P. D. Wroath, Rutherford Laboratory PLA Progress Report, 1963 (unpublished).
[c] C. J. Batty, R. S. Gilmore, and G. H. Stafford, Nucl. Phys. **51**, 255 (1964). This reference compares the current measurements to those of L. H. Johnston and Y. S. Tsai, Phys. Rev. **115**, 1293 (1959), but incorrectly quotes the probable errors of Johnston and Tsai as standard deviations. Thus, the quoted errors of Batty *et al.* are a factor of three, rather than two, better than the errors of Johnston and Tsai.
[d] C. J. Batty, G. H. Stafford, and R. Gilmore, Phys. Rev. Letters **2**, 109 (1962).

TABLE III. Results of the phase shift analyses of the 29 data with the higher angular momentum phases fixed at the CR21 values (see Table IV and text). The number of free, searched upon, phases is denoted by $N$. "Phase" indicates the phase shift just released from its CR21 value. $\chi^2$ is the least-squares error sum, $M$ is the number of degrees of freedom, and the $\chi^2$ ratio is $\chi^2/M$. The $\chi^2$ and $F$ probabilities are labeled $P_q$ and $P_f$.

| $N$ | Phase | $\chi^2$ | $M$ | $\chi^2$ ratio | $P_q$ | $P_f$ |
|---|---|---|---|---|---|---|
| 5 | $^1D_2$ | 35.8 | 24 | 1.49 | 0.08 | |
| 6 | $^3F_3$ | 22.5 | 23 | 0.98 | 0.50 | 0.00 |
| 7 | $\epsilon_2$ | 18.5 | 22 | 0.84 | 0.68 | 0.03 |
| 8 | $\epsilon_4$ | 18.5 | 21 | 0.88 | 0.63 | 0.80 |

Solution 2 lower angular momentum phases[1] were also tried with the present data set, the higher angular momentum phases being fixed at the one-pion-exchange values. Since this solution does not correspond to the usual models, the order of release of the phases is not obvious. The order on the basis of $\chi^2$ is shown in Table V, where it is seen that the new data have definitely ruled against solution 2 at 50 MeV.[6] An indication of this result has also been found by Batty and Perring.[7]

A number of models[1] were compared to the present data set; the new $\sigma(90°)$ datum contributed about 100 to $\chi^2$ for the more recent models. The only exception was CR21, which had a contribution to $\chi^2$ of 1.8 from this datum.

## IV. COMPARISON TO BATTY AND PERRING

Batty and Perring[7] have also made a phase-shift analysis of the present data: There are several differing points between their work and this. For instance, they did not consider the possibility of gross instability of the $^3F_3$ phase. Thus, they show considerably better definition of the phase shifts than may seem warranted on the basis of the present work. Again, their analysis

[5] P. Signell, N. R. Yoder, and J. E. Matos, Phys. Rev. **135**, B1128 (1964).
[6] The much larger $\chi^2$ of Solution 2 appears to be due mainly to $D(70°)$ and $A(23.5°)$.
[7] C. J. Batty and J. K. Perring, Rutherford High Energy Laboratory, NIRL/R/63, 1964 (to be published); also Nucl. Phys. (to be published).

TABLE IV. Nuclear bar phase shifts, in degrees, corresponding to the analyses shown in Table III. Phases not shown were at the corresponding CR21 values.

| $N$ | $^1S_0$ | $^3P_0$ | $^3P_1$ | $^3P_2$ | $\epsilon_2$ |
|---|---|---|---|---|---|
| 5 | 38.81±0.49 | 11.54±0.66 | −7.68±0.45 | 6.07±0.20 | |
| 6 | 38.68±0.39 | 9.96±0.76 | −8.19±0.39 | 6.18±0.18 | |
| 7 | 38.07±0.47 | 10.26±0.71 | −8.04±0.37 | 6.26±0.17 | −2.15±0.25 |
| CR21 | 38.27 | 10.48 | −8.87 | 6.30 | −1.58 |
| OPE | | 21.50 | −12.43 | 1.14 | −1.87 |
| $N$ | $^1D_2$ | $^3F_2$ | $^3F_3$ | $^3F_4$ | $\epsilon_4$ |
| 5 | 1.47±0.14 | | | | |
| 6 | 1.94±0.18 | | 0.43±0.34 | | |
| 7 | 2.32±0.22 | | 0.24±0.34 | | |
| CR21 | 1.76 | 0.28 | −0.73 | 0.08 | −0.21 |
| OPE | 1.15 | 0.38 | −0.79 | 0.07 | −0.21 |

was made at 51.65 MeV[8] without interpolation of the $A$ data.[8] This may be because the Proton Linear Accelerator (PLA) energy was not determined at the time their analysis was initiated. Use of the higher energy, however, would seem to negate to some extent the standard deviations achieved by the experimental group. Finally, an attempt was made here to duplicate Batty and Perring's data treatment and phase-shift analysis. The resulting phase shifts agreed with theirs to within their quoted errors, but the phase-shift standard deviations differed by as much as a factor of three. A check was made by fixing the phase with the largest discrepancy at three values in turn, with $\chi^2$ reminimized at each. A parabola was then drawn through the points; it predicted both the phase shift and standard deviation to be at the same values as had been obtained here by the usual method. It would seem unlikely, then, that the present calculation is in error.

## V. DISCUSSION

The $^3F_3$ phase shift for $N=6$, Table IV, is over three standard deviations from its one-pion-exchange value. One can not conceive of a model which would produce such a result. The tendency would be to discard $N=6$ and revert to $N=5$ as the preferred solution. Yet $\chi^2$ drops by more than a third in going from $N=5$ to 6. This *cannot be ignored*; it reflects on the consistency of the data. If one could blame a single datum, the situation might be recoverable; but that is not the case. One can seemingly conclude only that the phase shifts probably lie somewhere between the extremes shown in Table IV. Thus, the $^3P_0$ phase shift probably lies between 9.2 and 12.3°, but even that is far from certain. More precise definition of the phase shifts will have to await the results of future experiments.

Finally, it should be noted that the $^1D_2$ phase shift at 50 MeV is interesting from the standpoint of the Amati-Leader-Vitale (ALV) "$2\pi$ basic" cross-channel contributions.[9] If the ALV contributions are correct, it is not possible to obtain a $^1D_2$ phase larger than about 1.0° at 50 MeV, without a drastic departure from either the ALV or the boson exchange models for the $2\pi$ $s$- and $p$-wave contributions.[9]

TABLE V. As in Table III, but with the lower angular momentum phases of the type for solution 2, and the higher angular momentum phases at their OPE values.

| $N$ | Phase | $\chi^2$ | $M$ | $\chi^2$ ratio | $P_q$ | $P_f$ |
|---|---|---|---|---|---|---|
| 5 | $\epsilon_2$ | 76.5 | 24 | 3.19 | | |
| 6 | $^1D_2$ | 67.0 | 23 | 2.91 | | |
| 7 | $^1G_4$ | 47.5 | 22 | 2.16 | <0.03 | 0.00 |

## ACKNOWLEDGMENTS

Communications of data in advance of publication by A. Ashmore and R. C. Hanna were greatly appreciated. Most of the ideas and computer programs used here were developed in various collaborations with N. R. Yoder. The calculations were carried out in the Computation Center of the Pennsylvania State University and the Atomic Energy Commission Computation Center at New York University.

[8] J. K. Perring (private communication).

[9] J. W. Durso and P. Signell, Phys. Rev. 135, B1057 (1964).